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Life cycles of granular materials

D. MuirWood

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THE ROYAL SOCIETY

Life cycles of granular materials

By D. Muir Wood

Department of Civil Engineering, University of Bristol, Queen's Building, University Walk, Bristol BS8 1TR, UK

Granular materials occur in many geological and engineering situations. They can pass through distinct stages of existence separated by transitions of density, deformation rate and behaviour during which structure is alternately created or destroyed. Behaviour during one stage is dependent on the outcome of the previous stage.

Much geotechnical design is concerned with small deformations of materials which have been deposited by ice, water, air or human action and subjected to a history of tectonic and geomorphological forces. The till which has been continuously reworked by glacial action, or the sediment transported by water or air, becomes a firm foundation material but with an anisotropy of stiffness linked to the process of formation. However, earthquake loading can cause the same soils to liquefy and flow and some civil engineering applications involve large relative movements of soils and structural elements. As they are poured into a hopper, granular materials develop a structure which influences the development of stress on the walls of the hopper and the patterns of flow and loading during emptying.

A generic life cycle for granular materials is presented and its application to a number of different types of material is discussed. Some phenomena observed in the mechanical response of geotechnical granular materials are described.

Keywords: structure; deformation; flow; deposition; stiffness; soils

1. Introduction

Granular materials are responsible for many of the problems encountered in civil and chemical engineering. Materials of geological origin are encountered as foundation and fill materials in geotechnical engineering. Quite different materials may form the contents of silos or the raw ingredients or final products of many food and pharmaceutical processes. Geotechnical engineering usually has to make do with whatever granular materials are available, whereas, in process engineering for pharmaceuticals and foods, it is possible to design individual particles to give a desired overall behaviour. Efficient design of such materials requires understanding of the physical processes which control the behaviour and interaction of their constituent particles. Efficient design of applications requires understanding of the ways in which they flow, the ways in which groups of particles form, different sizes segregate and individual particles fracture.

Engineering soil mechanics is required to make predictions of the performance of geotechnical structures under static or dynamic loading using numerical models of soil behaviour. Many of the models that have been produced are empirically based. The sophistication of this modelling is not necessarily matched by confidence in the underlying physical principles.

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Understanding and controlling the behaviour of granular materials requires observation and modelling of the materials at various times during their continuing lives as they pass through more or less distinct stages separated by transitions during which their structure or fabric is alternately created and destroyed. In this paper, some characteristics of the behaviour of granular materials are related to the stage within the life cycle of the material at which they occur. The behaviour at any one stage may often depend on the way in which the material reached that stage, so that understanding of the nature of the life cycle is important if response is to be successfully predicted. The concept of life cycles will be used here to draw analogies between different types of granular material and hence identify possible common features.

Many of the examples will be taken from geotechnical engineering and soil mechanics, reflecting the research background of the author. Examples of common problems in geotechnical engineering are shown in figure 1, with an indication of the typical magnitude of the deformations of the soils involved. However, many of the issues relating to the modelling of soils are relevant to other situations where granular materials are encountered. Different situations demand understanding of different key features of mechanical response.

2. Generic life cycle

A generic life cycle for granular materials is shown in figure 2, in which porosity (ratio of volume of pore space to total volume of granular material and hence inversely related to density) is plotted against time (or sequence). Usually the highest porosities (lowest densities) will be associated with transport and flow and hence (relatively) high velocities or rates of deformation. The cycle is not intended to have any particular end or beginning: during the life of any particular material the sequence may well repeat, as indicated by the arrow in figure 2 and as described in the examples below. Different granular materials may enter or leave the cycle at different stages: characterization of their mechanical behaviour may be required at quite distinct points within their particular cycle. The diagram is purely schematic and neither axis should be thought of as implying any regular, still less linear, scale.

Typical stages in the life cycle might include the following.

- A Dilute dispersed flow with minimal interparticle contact: the highest porosities are likely to be associated with free fall through, or transport by, a surrounding fluid (for example, water or air).
- B Inertial flows with particle interaction; for example, bed transport by water or wind.
- C Non-inertial decelerating flow leading to sedimentation or deposition.
- D Static undisturbed continuous state.
- E Recoverable elastic response to stress change.
- F Irrecoverable plastic deformation or loss of strength under larger changes of stress.
- G Discontinuous heterogeneous deformation with localized concentrations of strain and variations in density.
- H General dense concentrated flow with strong particle interaction.

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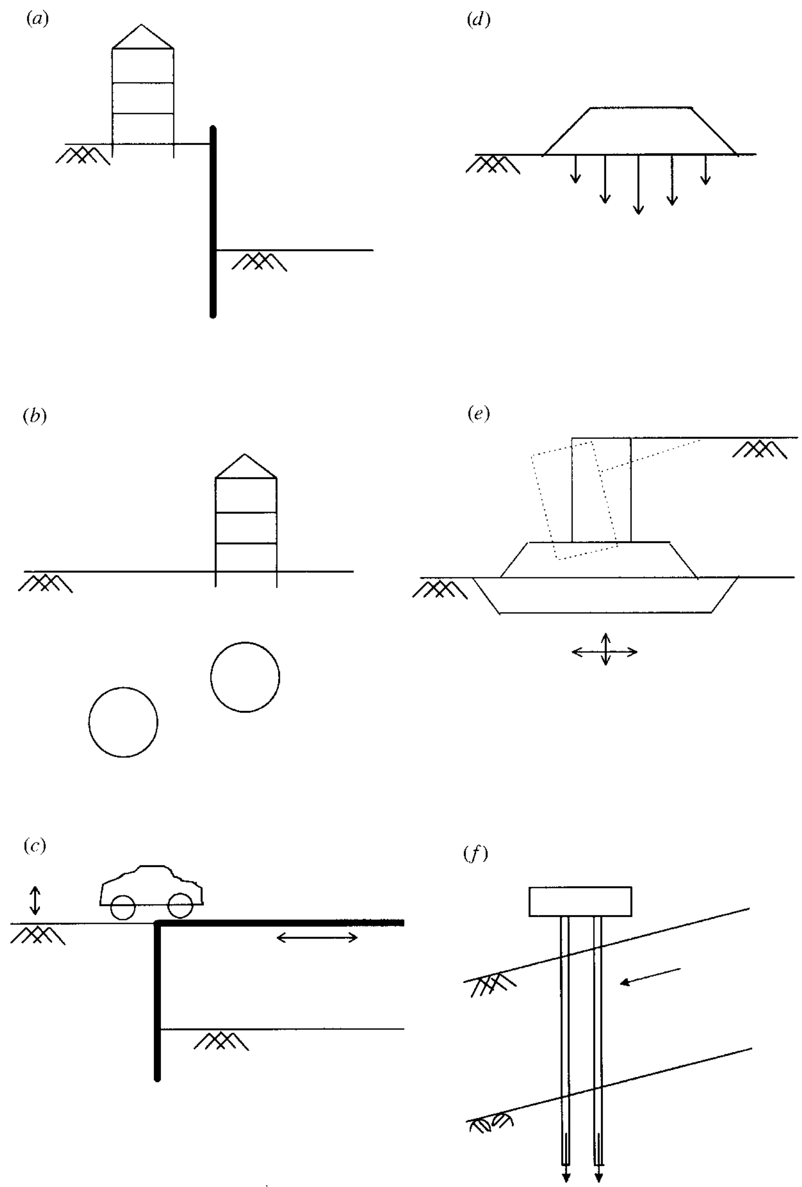

Figure 1. Typical geotechnical problems. (a) Deep excavation beside existing building: small deformations caused by movement towards excavation. (b) Tunnels beneath existing building: small deformations caused by ground loss during tunnelling. (c) Integral bridge abutment: small deformations but repeated loading both from traffic and thermal expansion of bridge deck. (d) Embankment on soft alluvial ground: medium deformations caused by plastic deformation of foundation soils. (e) Failure of quay wall through earthquake induced liquefaction: large deformations at failure (final position of wall shown with dotted line) produced by horizontal and vertical accelerations of foundation soils (see Inagaki *et al*. 1996). (f) Structure supported on piles to rock through unstable slope or creeping landfill: large deformations caused by downslope movements of artificial or natural materials.

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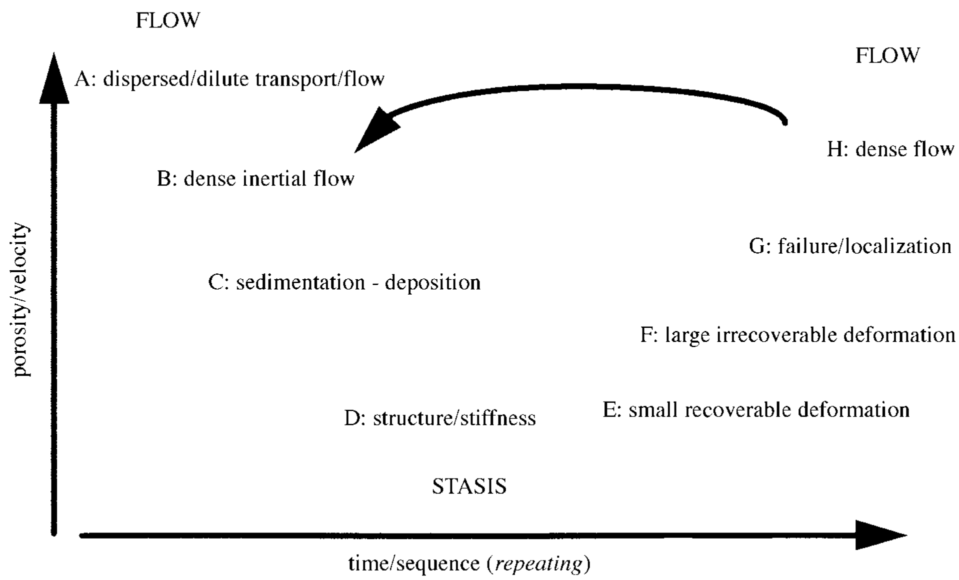

Figure 2. Life cycle for granular materials.

Some specific examples may help to illustrate these ideas through their application to civil engineering, bulk solids handling and geological situations. The general picture is also relevant to flows of people or movements of traffic (Roberts 1994; Gavrilov 1997): the transitions between flow and stasis are all too familiar and the contrast between the dilute flows on remote Highland roads and the dense flows of central London is obvious.

3. Silos

The process of filling and emptying a silo fits obviously into the life cycle. The dilute flow of the granular material into the silo is flow through, rather than transport by, a surrounding fluid. As the silo fills, the large deformation process of material flow (stages A, B, C) becomes the small deformation response of the material already in the silo (stages D, E, F) controlled by the material structure—defined as the orientation and distribution of particles together with the forces between adjacent particles (Lambe & Whitman 1969)—formed by the deceleration of the input flow (stage C). The particle arrangement that develops on filling is dependent on the rate of filling, particle shape, particle surface characteristics and the geometric details of the filling operation.

When the silo outlet is opened, a mechanism of deformation develops in initially stationary material (stage D to H and back to A); some of the material remains stationary, some is flowing, some is undergoing transition from stasis to dense flow and through the outlet to dilute flow. The particle alignments trapped in the granular material at the end of the filling affect the flow process itself. The interaction between the dense flow and the unflowing material in the silo controls the stresses that are experienced on the walls of the silo. On closure of the outlet, the flowing material is brought to a halt (stages B, C).

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4. Sedimentary soils formed by deposition from suspension

Many materials begin with a stage of transport and dilute flow in which little or no particle interaction occurs and the effective porosity is very high. Many of the sediments that are today subjected to engineering loads in geotechnical structures, for which the deformation response under load is required (stage D to E or F: figure 2), were carried into lakes or oceans by river sediment transport (stage A to B to C), having previously been eroded from rocks or existing sediments by action of wind, weather or temperature, or mechanical action. As the speed of the transporting medium reduces, the granular material is deposited and a sediment develops in which individual particles can interact through mechanical contact or molecular forces. The material starts to act as a continuum and to transmit loads and stresses over distances which may be large by comparison with the dimensions of individual particles.

Oceanic turbidites can develop as a result of failure of sediments on continental slopes (stage D moving to H and then to A or B), giving rise to flows which, as they become more energetic, give rise to single-phase suspensions and particle separated suspensions (stages A or B), which then settle on the deep ocean floor to become the dominant deep ocean sedimentary soils (stage C to D). Erosion of sediments by dredging or by increasing fluid velocity of either air or water, changes continuous soil back to dilute flow: a granular equivalent of sublimation.

Compaction of stable sediments (D) proceeds under the weight of the sediments or of other loads. The continuous granular material develops further structure and stiffness properties which control the deformations that develop under changing external loads. For geological deposits, there may be significant changes in these loads through tectonic action, glaciation or geomorphological processes which influence the structure of the material before it starts to be of engineering interest. For many materials, the time spent in this stage may be very long and there is potential for changes in the interparticle bonds.

The chemical environment in which the sediment is formed can have a major influence on the properties of the sediment. Deposition in a carbonate rich fluid can lead to cementation between particles, a bonding that may later break down as the soil is loaded (Cuccovillo & Coop 1997). Clays formed in marine environments often have a much more open structure than those formed in fresh water: isostatic uplift leaves the structure in a metastable state which can be easily disrupted with the disastrous consequences of the quick clay flow slides that are common in Scandinavia and Eastern Canada (see, for example, Bjerrum 1971).

Sediments deposited slowly over areas of large lateral extent and not subjected to major tectonic or other disturbances may be expected to have a cross-anisotropic (hexagonal) symmetry of structure and mechanical characteristics. Such a symmetry justifies the standard practice of taking cylindrical samples from the ground from vertical boreholes and then testing them in so-called triaxial apparatuses which, with independent control of radial cell pressure and axial stress, preserve and rely upon the same symmetry. The instrumentation used in even the most sophisticated triaxial installation cannot discriminate the anisotropic/asymmetric deformation and stress distribution characteristics that would develop in samples which lack this symmetry.

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5. Glacial till soils

The life cycle for glacial till soils becomes of interest at a stage when the material is already undergoing dense flow (stage H or B/C in figure 2). The material that is a glacial till is the same material as it is deformed and flowing slowly beneath a glacier as when it is subsequently subjected to engineering loading: a continuity of modelling might be sought.

A moving glacier erodes underlying rocks and soils and slides on a layer of continuously sheared till of varying thickness at rates of shear strain which may be high on a geological scale and continue for long periods. Many glacial till soils form as a consequence of very large strain deformation of pre-existing sediments, reflecting an evolution through stages D to H. Localization of deformations within the shearing material may lead to particle breakage, mixing and homogenization or alternatively to segregation and sorting. Deformation patterns and rates are strongly dependent on the drainage and water pressure regimes. On retreat of the glacier, the till reverts to stage D, with little opportunity for change of particle arrangement and structure during the slow deceleration of flow, leaving a deposit of till whose properties are conditioned by its history, which determines how it responds to later applied stresses during engineering construction (stages E, F).

The models that have been successfully applied to sedimentary soils, for which the history of stresses may seem reasonably well defined, have been less successful for glacial tills for which the history is more complex. When it comes to determining and describing the *in situ* deformation characteristics of glacial soils, it may be inappropriate to assume simplifying symmetries. There is no obvious reason why the material should be cross-anisotropic (with a vertical axis of symmetry) in its stiffness and structure when the previous shearing by the overlying or adjacent glacier has been subhorizontal.

The range of particle sizes, and range of particle mineralogies, is greater in glacial till soils than in soils formed by other sedimentary processes. Glacial soils often have significant amounts of clay mineral present as well as more or less inert, usually larger, particles of stable rock minerals. The mechanical behaviour of such transitional soils combines features of the behaviour of clays, where volume change is resisted by the slow rate at which pore water can migrate through the low permeability clay, and features of the behaviour of sands, where direct mechanical interaction of hard particles has a dominant effect especially on the desire for volume change.

Tests on an artificial soil formed by adding increasing amounts of coarse (2 mm) sand to clay have shown (Kumar & Muir Wood 1997) that the presence of the sand has no effect on the mechanical response of the mixture until the proportion by weight of mineral exceeds *ca.* 65%; all volumetric effects are controlled by the clay/water phase. An example is shown in figure 3: drained triaxial compression tests with constant radial stress on samples of the kaolin–coarse sand mixtures isotropically compressed to 400 kPa and then unloaded to 100 kPa. The stress–strain response (figure 3*a*) is unaffected by the addition of sand until the proportion of sand by weight of mineral reaches *ca.* 70%. The volumetric response (figure 3*b*) is similarly unaffected by the presence of the sand, provided that it is expressed in terms of the volumetric strain occurring in the clay–water matrix: the clay volumetric strain.

A simple calculation based on uniformly distributed spherical particles suggests that the separation of the particle centres is of the order of 1.2 times the parti-

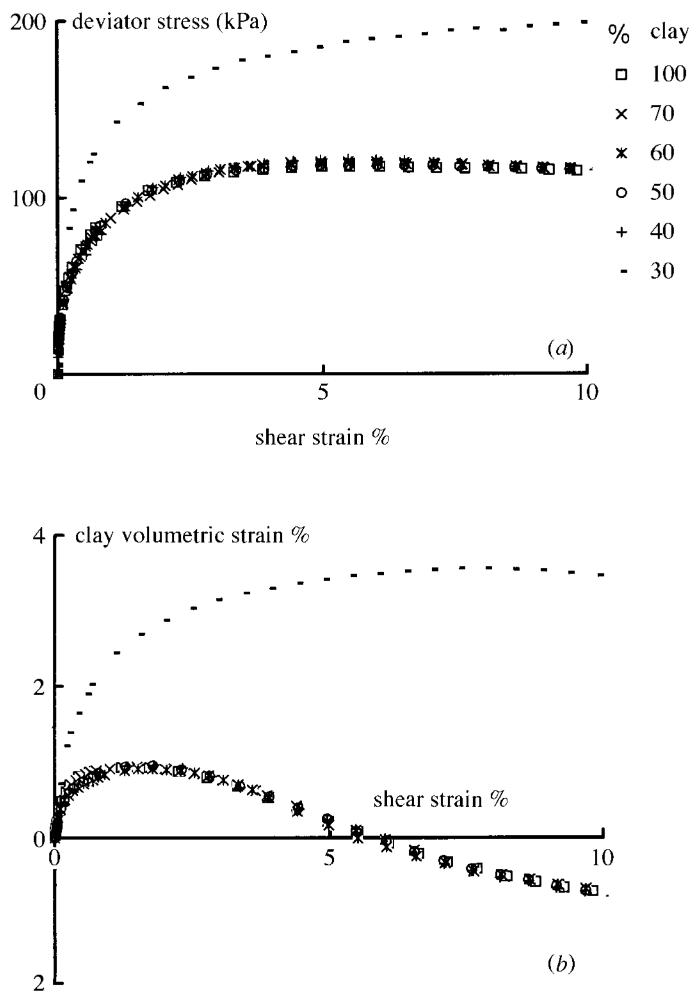

Figure 3. Drained triaxial compression of overconsolidated mixtures of kaolin and coarse sand: (*a*) shear stress and shear strain; (*b*) clay volumetric strain and shear strain (after Kumar & Muir Wood 1997) (figures indicate weight of clay as proportion of mineral in mixture).

cle diameter, on average, before detectable interaction of the large particles occurs. Until then, the sand acts merely as a filler, playing no mechanical part. Modelling approaches based on mixture theory would suggest a steady change in response as the sand content increases. However, there is a question about what sand property the property of the mixture should be tending towards. Clearly, the strength of the sand mineral itself is not directly relevant at the stress levels of these tests even for pure sand (although when the sand particles in pure sands do come into contact, the contact forces may be high enough to cause breakage of asperities). The ratio by volume of sand particles to overall volume (granular volume fraction) at the observed transition is *ca.* 42%, so that the porosity of the sand component (treating the clay–water matrix as filler) is very much greater than can ever be obtained in pure sands unless these are in a state of somewhat dilute rapid transported flow in which particle interaction occurs by occasional collision rather than by permanent contact.

6. Engineering fills

Many civil engineering projects require the placement of fill material to form foundations for roads or buildings, embankments, dams or backfill behind retaining structures. The cycles of these materials bear some resemblance to those of glacial till soils, but the transport is by human intervention and mechanical means. Such materials are usually prepared and dumped in such a way as to eliminate any pre-existing structure, but subsequent mechanical compaction using rollers and vibration will lock in stresses and deformations which control the future response. Again, there are no obvious symmetries emerging from the process of formation of the fill which can assist in the description of what can be expected to be an anisotropic structure with consequent anisotropic stiffness and deformation properties.

Engineered fills can be chosen to give certain required mechanical characteristics: good drainage characteristics, appropriate stiffness, impermeability for cores of dams: these are 'designer' soils. At the other extreme are materials found in landfills which are completely unengineered fills. For these, composition and homogeneity are unknown and the appropriate modelling strategy required to design foundations which sit on or pass through the fill onto competent underlying ground (figure 1*f*) is uncertain. There are similarities between the unknown features of this situation and the loading that the piled foundations of offshore structures may have to resist from the flow of submarine sediment failures or the loadings on piled foundations of land structures from more limited ground flows caused by natural (landslides) or man-made loading (adjacent embankments).

When placed behind a retaining structure, the stiffness characteristics of the fill play a major role in controlling the magnitude and distribution of the loads on, and hence stresses in, the wall. It is this ground–structure interaction which will control the structural design of the wall itself. Although many retaining structures are subjected to more or less constant loadings, the increasing use of integral bridges—dispensing with bearings between deck and supports—produces abutments which are required to accept by movement (or stiff resistance) the effects of daily and seasonal thermal expansion of the bridge structure (figure 1*c*) (Card & Carder 1993). An efficient design may be obtained by using a flexible wall to carry the vertical load from the bridge with little flexural restraint. But what are the horizontal pressures that will develop on this wall as it is cycled over a lifetime of 50 years? Will the steady 'dither' provided by the traffic loading help to ease the stresses that may build up or merely densify the fill and increase its stiffness?

7. Modelling

The major common processes that have to be addressed in many different applications are: (a) the transition from stasis to flow; (b) the processes of flow; and (c) the transition from flow to stasis and the subsequent mechanical response. Each of these processes will be influenced by particle shape, size distribution, systematic orientation and inhomogeneity of present packing. Each of these processes can be approached through continuum or discrete particle modelling, modelling the material structure at micro-, meso- or macro-scale.

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The individual stages in the life cycle are often treated independently for the purposes of analysis, but the response during one stage is dependent on the outcome of the previous episode and some continuity of behaviour might be anticipated. One reason for the subdivision of modelling is that the deformations in successive episodes through the life of a single granular material are frequently alternately large and small with phases of flow separated by stasis, and with history—in the form of material structure—being alternately created and destroyed, so that the natures of the analytical approaches that might be suggested are different. The separate disciplines of earth sciences, civil engineering soil mechanics, solids handling and process engineering have developed independent modelling techniques. There are common features, but each discipline sees special characteristics which are of secondary importance to others.

Estimations of the deformation response of geotechnical structures to applied loads are required (figure 1). The models that are employed for this purpose are expected to make some allowance for the geological history of the granular material. A model is a basis for extrapolation: from the single particle to the assembly; or from the behaviour observed in 'single-element' laboratory tests, performed under rather restricted conditions of stress and strain, to the completely general stress and strain conditions encountered in practice. The security of the extrapolation will be enhanced if the physical basis for the model itself is secure. With an appropriate model, an observation interpreted at the continuum level can be analysed for its implications at the particulate level.

The engineering approach to the modelling of granular materials has traditionally treated them as continua and has sought, through observation of behaviour in laboratory single-element tests, laboratory models and field or full-scale measurements, to construct appropriate constitutive frameworks in terms of the continuum quantities of stress and strain. There are a number of competing constitutive frameworks, such as hypoplasticity (Gudehus 1996), bounding surface/kinematic hardening plasticity (Manzari & Dafalias 1997; Gajo & Muir Wood 1997), endochronics/strain space modelling (Valanis & Read 1982), each of which has a certain mathematical elegance but none has yet been able to match all aspects of soil behaviour.

Granular materials consist of individual particles, which for clays may be packets of molecules, which interact mechanically (or through electrostatic molecular interactions), transmitting loads through the grain contacts and possibly fracturing in the process. The idea of stress is clearly useful at a scale which is large by comparison with the size of individual particles, but may not be so useful when interparticle action itself is being considered. An alternative approach to describing the mechanics of granular materials therefore starts from the mechanics and physics of particle interactions and attempts to develop constitutive insights from analysis of particulate assemblies. Two-dimensional simulations can be misleading in that, in comparison with the real three-dimensional world, they may exaggerate certain features and fail to capture others. However, they do have the advantage of providing clear visualizations which enable significant features to be identified and quantified. Three-dimensional simulations are quite feasible and the size of the problem to be analysed and the number of particles that can be included are in principle limited only by available computer power and time. However, it is evident that most problems of engineering interest involve a greater number of particles than can presently be included in numerical analysis.

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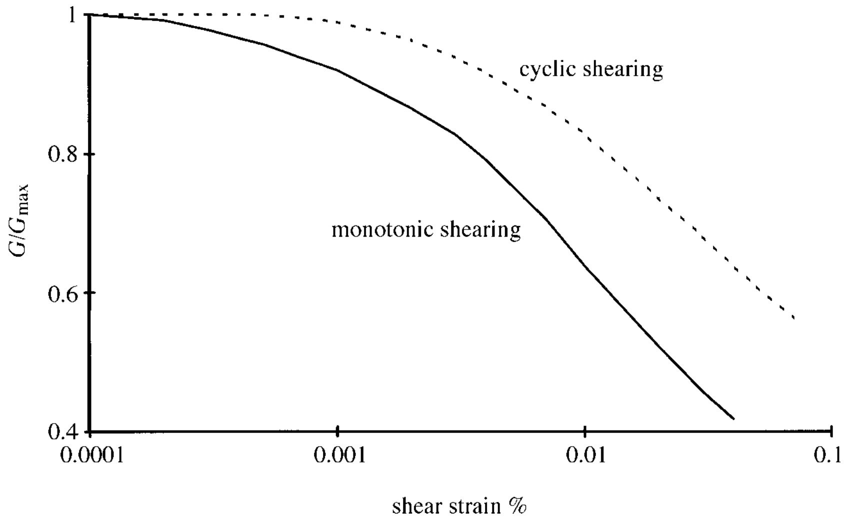

Figure 4. Variation of shear modulus G with shear strain for monotonic and cyclic torsional shear of Toyoura sand (derived from data of LoPresti *et al.* 1997) ($G_{\max}$ is value of shear modulus at extremely small strain).

8. Small deformation problems

For most geotechnical purposes, the materials of interest are in state D and the modelling concern is the response under changes in stress E and F. For many geotechnical structures under working loads, the deformations are small (figure 1*a*,*b*,*c*). The response of the material under small deformations is controlled by the arrangement of individual particles and the arrangement of the particles is not greatly affected by the changes in loading. As precision of measurement of deformations of soil samples has improved over the past two decades, it has become clear that the stiffness (the stress change required to produce a given change in strain) falls off rapidly as the strain level increases (for example, figure 4 (after LoPresti *et al.* 1997)). The region of stress or strain in which soils might be described as truly elastic, producing an entirely recoverable response to perturbations with shear stiffness $G_{\max}$, is very small, corresponding to shear strains of the order of $1 \times 10^{-4}\%$, and typical engineering applications take the soil well beyond this elastic region at least close to a foundation or structure so that the mobilized stiffness falls to a reduced value G. Characterization of the stiffness $G_{\max}$ at very small strain levels provides an anchor on which to attach the subsequent stress–strain response and is directly relevant to the response of the 'far' field soil which provides containment to the more extensively deforming material.

At the macroscopic level, laboratory experiments show that the response of granular materials is nonlinear and inelastic even at extremely small strains. Granular materials have a limiting frictional strength, so the relationship between stress state and deformation is dependent on the initial state of the material. In situations where the granular material interacts with a stiff boundary—fill behind a sheet pile wall, contents within stiff walls of a hopper—the contact stresses are dependent on the stiffness that is immediately developed in the granular material.

Laboratory geophysical testing using piezoceramic bender elements demonstrates

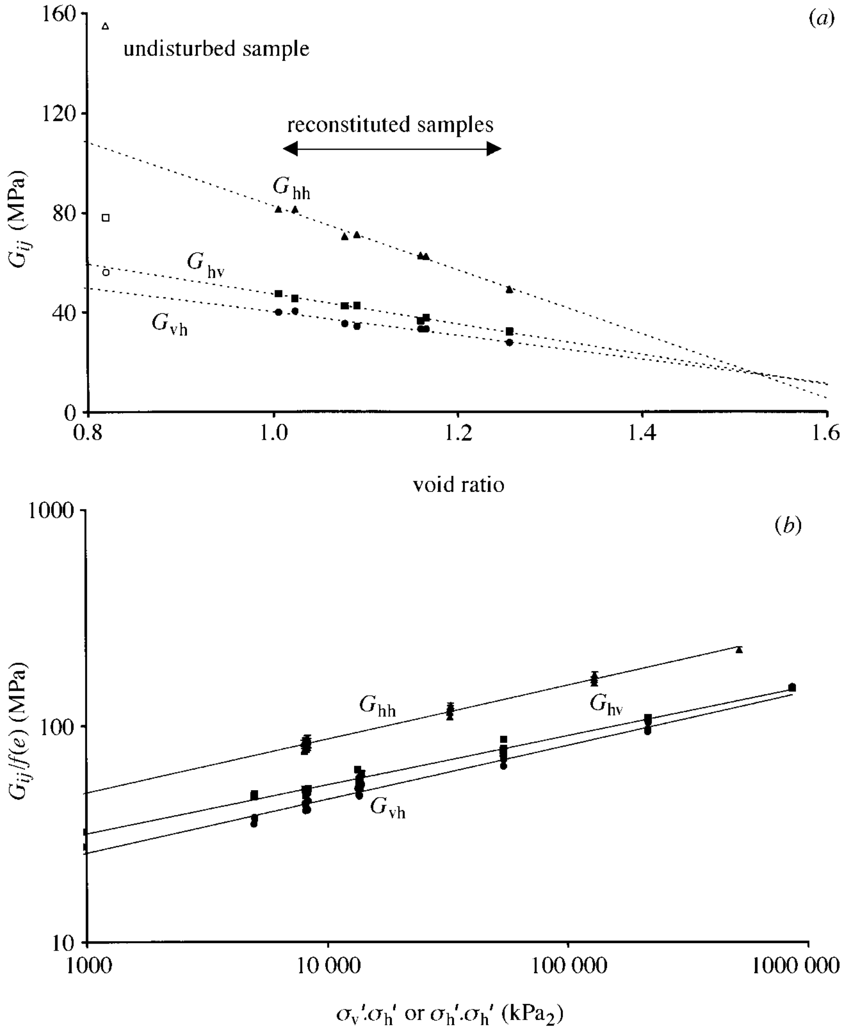

Figure 5. Small strain shear moduli of Gault clay: (*a*) as function of void ratio at constant isotropic stress state; (*b*) as function of stresses at anisotropic stress states (after Pennington *et al.* (1997)). ($G_{\rm hh}$, $G_{\rm hv}$, $G_{\rm vh}$ are shear moduli measured with different directions of polarization and propagation in laboratory seismic tests. In (*b*), the moduli have been normalized for the void ratio effects on shear modulus seen in (*a*) and these normalized values are plotted against $\sigma_{\rm v}'\sigma_{\rm h}'$ or $\sigma_{\rm h}'\sigma_{\rm h}'$, depending on the directions of polarization and propagation of the shear waves.)

that the stiffness of granular materials (deduced from the characteristics of shear wave transmission) is anisotropic (for example, figure 5 (data from Pennington *et al.* 1997)) at the very low strains imposed by the bender elements, and that the anisotropy depends on the macroscopic stress conditions to which the material is subjected even without any significant rearrangement of particle structures. Anisotropy is significant because of the effect that it has on attracting stress, on propagation of shear waves and hence on the seismic response of granular materials, and on far field response to construction perturbations (see, for example, Simpson *et al.* 1996).

Many laboratory studies on many soils have shown that small strain shear stiffness can be described by an empirical expression of the form (see, for example, Roesler 1979; Bellotti *et al*. 1996; Shibuya *et al*. 1997)

$$G_{0ij} = S_{ij} f(e) p_{\mathrm{a}}^{1-ni-nj} \sigma_i^{ni} \sigma_j^{nj}, \tag{8.1}$$

where the directions i and j of polarization and propagation, respectively, of the shear wave coincide with the principal axes of stress and of anisotropy of the material, and G_{0ij} is the shear stiffness deduced from shear waves propagating in the i direction and polarized in the j direction. The factor S_{ij} describes the influence of the structure of the soil on G_{0ij}, and the function $f(e)$ describes the effect of change in void ratio. Experimental evidence suggests that ni and nj may be similar in magnitude and have values around 0.25 for sands and clays (figure 5*b*). However, a wide range of values has been reported and, in particular, it appears that smaller values are obtained from tests on undisturbed samples of natural clays (Pennington *et al*. 1997) (although Butcher & Powell (1995) report very much higher values deduced from *in situ* geophysical measurements on several clay sites). An exactly similar relationship has been used successfully to describe uniaxial confined stiffness, deduced from propagation of compression waves. This stiffness depends only on the principal stress in the direction of propagation to a power of the order of 0.5.

Expression (8.1) suggests that the stiffness depends on the principal stresses in the directions of polarization and propagation but not on the third principal stress. The introduction of atmospheric pressure p_{a} as a normalizing stress is not a rational choice: it would be more logical to link the stiffness to some material characteristic such as the shear stiffness or strength of the mineral forming the soil particles (Bolton & McDowell 1997) or 'granulate hardness' (Gudehus 1996).

The structure term S_{ij} in (8.1) is assumed to be somehow independent of stress state. Logically, the structure of the soil is expected to change as the soil suffers irreversible plastic deformation and this has been seen to occur at very small strain levels. It is then not clear what the separate effects are that S_{ij} and the principal stress terms are describing and what the range of application of (8.1) should be with constant values of S_{ij}. A particulate approach might be helpful, in that S_{ij} would be expected to encapsulate the contact arrangement (coordination number, see for example, Chen & Ishibashi 1990), whereas the stress terms indicate the forces on those contacts that are exercised by the passage of the shear or compression wave.

9. Dense flow

As the deformation applied to a granular mass is increased, radiographic observations show internal density variations with a pattern that may have some regularity at a meso-scale (figure 6). The bands of localized dilation thus revealed are not obviously linked with discontinuous boundary displacement (although they may be associated with strain gradients), and can be described as continuum localizations (Desrues *et al*. 1985, 1996; Muir Wood & Stone 1994). The width and spacing of the zones of dilation have dimensions controlled by particle size—10–20 particle diameters are typically quoted (Roscoe 1970; Scarpelli & Wood 1982)—and there is a scale of modelling with dimensions less than this at which this heterogeneity must be incorporated. However, geotechnical structures will have typical dimensions several orders of magnitude greater than the size of individual particles and there may be another scale, with resolution not less than, say, 500 particle diameters, at which the

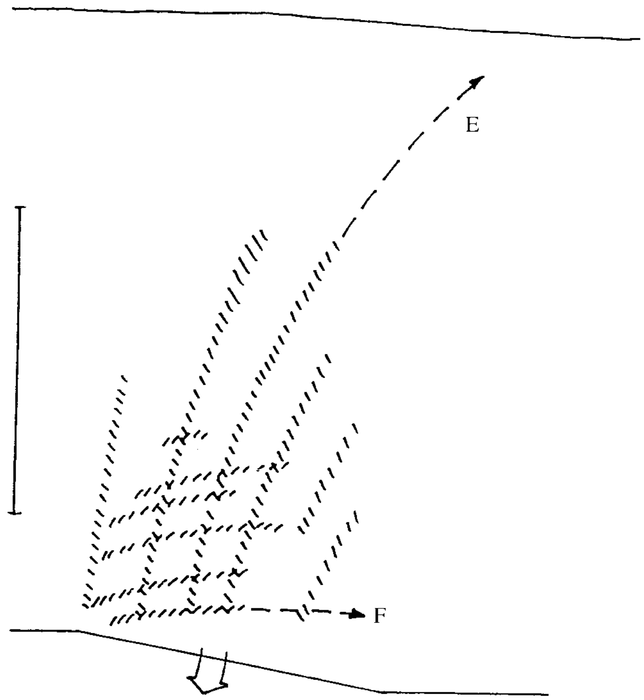

Figure 6. Continuum localizations seen in model tests on dense sand: part of base of model rotated (sketch of radiograph, after Muir Wood & Stone (1996)). (Hatched areas show regions of density decrease resulting from shear-induced dilation at small to medium strain. Dashed lines E and F show failure zones that develop after large boundary displacement. Marker bar represents 100 mm, particle size of sand *ca.* 0.85 mm.)

behaviour can again be treated as homogeneous. What is the macroscopic behaviour of material containing such mesoscopic heterogeneities? What is the role of small laboratory 'single-element' tests in investigating and characterizing this macroscopic behaviour? What modelling strategies can be justified from physical considerations?

Dense flow comes at an extreme of mechanical behaviour. If a granular material is sheared to very large deformations, then it is reasonable to suppose that it will forget any initial structures that it may have had. Many constitutive models assume that a regular class of ultimate flow states (combinations of stress states and particle arrangements and densities) can be defined to which soil elements will ultimately tend with continued monotonic shearing (see, for example, Gudehus 1996). Models such as Cam clay (Roscoe & Burland 1968), which have enjoyed reasonable success for modelling of medium deformation problems such as embankment loading on soft soils (figure 1*d*), assume that clays tend to critical states at which shearing continues without change in stresses or density. Are these states of perfect plasticity real at a scale that is helpful and how can such states be characterized?

Conventional testing apparatuses are not able to impose infinite shear strains on homogeneous samples. Ring shear apparatuses have been used to impose large relative displacements across soil samples. In clayey soils, such tests are expected to lead to the formation of more or less continuous thin shear zones within the sample, and in the vicinity of such shear zones the clay particles become aligned

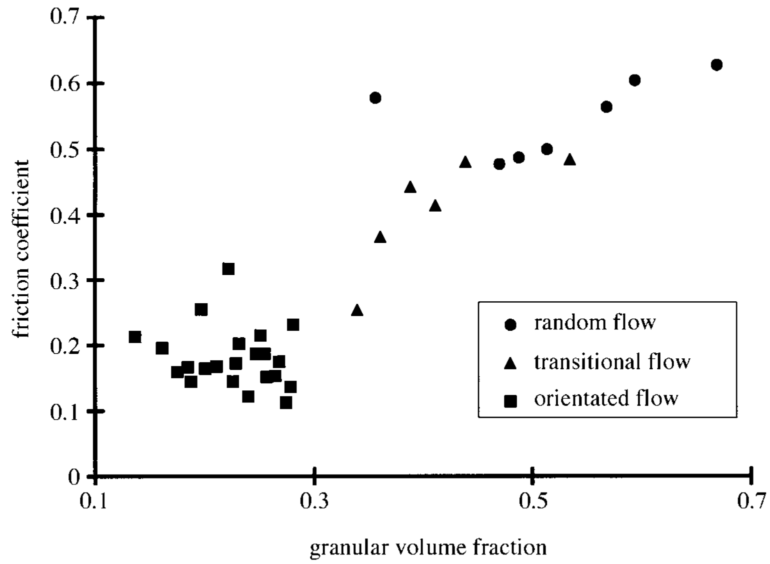

Figure 7. Dependence of failure mode in ring shear tests on granular volume fraction (after Lupini *et al*. (1981)). (In orientated flow, polished surfaces form in the clay matrix; in random or turbulent flow, a failure region rather than a failure surface is seen.)

parallel to the sliding surface. The frictional strength that can be mobilized on such a polished surface is much lower than for a randomly orientated material. In mixtures of clay and sand, Lupini *et al*. (1981) show that continuous failure surfaces may form if the spacing of the hard non-clay particles is large enough to provide kinematic freedom (figure 7) which requires granular volume fraction below *ca.* 30%, whereas a completely random flow requires granular volume fraction above 45%. These figures are similar to those for which transitions of response were observed by Kumar & Muir Wood (1997) in the prefailure and volumetric response of clay–sand mixtures (and also similar to those quoted by Hallworth & Huppert (1997) for transitions in regimes of behaviour observed in experiments on flow of particle suspensions). A random flow may not be an appropriate asymptote for materials in which reorientation of particles to assist flow can occur.

In dense sands the failure surfaces that develop in homogeneous single-element tests are visible to the eye: mathematically they can be seen as a consequence of a certain character of continuum constitutive response, of which non-associated plastic flow and strain softening are key ingredients (Vermeer 1982; Vardoulakis 1978). Once a shear band has initiated, deformations become concentrated there and it is only in this shearing region that continuing softening to critical states and development of a constant volume flow structure actually occurs (Vardoulakis 1978; Finno *et al*. 1997): the initially homogeneous sample is now clearly heterogeneous with two essentially undeforming blocks separated by a region of intense shearing. The density and stress conditions at which a material will flow at constant volume under conditions of axisymmetric compression and extension might be expected to differ from those observed in conditions of plane strain. However, Desrues *et al*. (1996) show that in triaxial tests on Hostun sand ($d_{50} = 0.32$ mm), with frictionless end platens and diameter and height equal to 100 mm (a test configuration which is regarded as optimal for measurement of deformation and strength characteristics), an orderly network of localized regions of density change is observed. An overall axisymmetric deformation response seems to be a summation of a series of plane mechanisms: the natural variability of the structure of the sand gives enough kinematic freedom for this to occur.

Flow states can develop at elevated stress levels, for example, in glacial till under a glacier, or at very low stresses in material flowing from a hopper, in a flowslide (such as Aberfan), or liquefying in an earthquake. Are there physical bases for defining the properties of a class of flow structures? If asymptotic flow states exist, towards which the material tends if sheared monotonically, then in those states the flow must occur at constant volume. The necessary thickness of a zone of granular material for such a flow structure to exist is typically quoted as *ca.* 10–20 particles (see, for example, Bridgwater 1980). If kinematic constraints prevent the formation of such a zone, then stresses will build up and produce particle breakage in order to reduce the required size of flow zone to an available level. Sassa *et al*. (1996) suggest that such particle breakage can turn a dense material (which would not normally be expected to liquefy since on shearing it would try to dilate and generate negative pore water pressures) into a loose material thus creating a changed mechanical environment. The possibility of particle breakage makes the testing and modelling of granular materials difficult. The definition of an appropriate reference material or state is complicated.

Liquefaction is an important natural and engineering process. Loose masses of granular materials liquefy through generation of pore pressures when subjected to earthquake or other forms of loading (stage D moving to G/H and thence to B). Reduced effective stress through the generation of pore pressures results in an inability to sustain shear stresses in a frictional material. In natural settings, rapid pore water pressure build-up in sediments during earthquakes can produce widespread hazardous and catastrophic liquefaction. The ground loses strength leading to foundation failures and flow sliding which may impose loads on engineering structures (figure 1*e*), such as the Kobe quay walls (Inagaki *et al*. 1996).

What are the macroscopic mechanical properties of the flowing material? These are required if the consequences of liquefaction are to be estimated. Since the material is flowing, it might seem appropriate to describe it as a viscous material for the purposes of estimating the lateral spread that will occur during and after an earthquake: Yashima *et al*. (1997) use a viscoplastic model with a threshold viscous yield stress. For description of landslide movement, Gray (1997) uses an analysis based on limiting stresses governed by considerations of frictional strength, whereas Vulliet (1995) uses a nonlinear viscous model. Can a viscous treatment of sand particles surrounded by water be justified physically, especially when viscous models are not obvious candidates for describing pre-flow behaviour? Many time-dependent aspects of macroscopic behaviour can be linked to rates of pore fluid diffusion, perhaps at the scale of particle clusters, which will be low if the flowing material contains a sufficient proportion of fine (especially clay mineral) particles.

This description of the flowing material is the middle of three modelling issues connected to liquefaction or flow. What is the mechanical route by which the state of flow is reached? Is it appropriate to use the same models for describing the small deformation behaviour of the material and for the estimation of the amount of earthquake shaking that is needed to produce the catastrophe of liquefaction (Arulanan-

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dan & Scott 1993)? What happens when the flow stops and soil redevelops a static structure? How will this stationary material behave in future? This is relevant to understanding the properties of sandy soils which have experienced such natural effects in their geological past (see, for example, Nichols *et al*. 1994). The potential of a deposit to develop liquefaction is dependent on the initial structure of the soil. When the pore pressures dissipate after the liquefaction event, the material is left with a new structure (stage C to D). Small deformations are incurred in the attainment of this new equilibrium but the resulting structure of the soil controls the future susceptibility to liquefaction and response to loading.

10. Conclusion

Geotechnical engineering requires answers to problems of geotechnical design. Geotechnical design has long worked from ultimate limit state calculations of collapse (in which sufficient soil is assumed to be in a state of swept out memory that a failure mechanism can develop) combined with load factors and experience to produce expected acceptable deformations. Many of the problems now encountered in geotechnical engineering relate to rational direct calculation of deformations for which a more sophisticated class of model is required. ‘Sophisticated’ does not necessarily mean complex, but the key to appropriate complexity is the correct identification of those events in the history of the soil and those physical characteristics of the soil particles themselves and their interactions that are really important.

At the medium strain level, when some of the detail of the initial structure of the material has been released and before asymptotic flow states become dominant, the estimates of deformation in geotechnical boundary value problems made using existing models are very often adequate. The quality of predictions is less secure at the low strain level required by consideration of soil–structure interaction or when the soil is in a state of flow.

Development of soil models has been broadly empirical with more or less arbitrary mathematical assumptions being introduced inspired by observation of element tests and physical models. Effective models for practical use must be based on appropriate simplifications and descriptions of the physics of the granular material so that they can be defined in terms of a finite number of physically plausible parameters. Such models will be able to point the way to appropriate laboratory or field testing procedures for determination of these parameters whose influence on predicted response should desirably be sufficiently orthogonal that clearly unique values of the parameters can be determined.

Grand universal theories of mechanical behaviour of granular materials covering all regimes of response are probably not only unrealizable but also undesirable: unrealizable because of the vast number of degrees of freedom that a particulate material inevitably possesses; undesirable because the unwieldy complexity of the model will mask any underlying simplicity that may be used to define models which are relevant to particular limited regimes. Understanding the life cycles of the granular materials can provide a framework for exchanging information between different applications of modelling of such materials and help to reveal likely successful modelling strategies.

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References

Arulanandan, K. & Scott, R. F. (eds) 1993 *Verification of numerical procedures for the analysis of soil liquefaction problems*. Rotterdam: Balkema.

Bellotti, R., Jamiolkowski, M., LoPresti, D. C. F. & O'Neill, D. A. 1996 Anisotropy of small strain stiffness in Ticino sand. *Géotechnique* **46**, 115–131.

Bjerrum, L. 1971 *Kvikkleireskred*. Publication 89, Norwegian Geotechnical Institute.

Bolton, M. D. & McDowell, G. R. 1997 Clastic mechanics. In *Mechanics of granular and porous materials* (ed. N. A. Fleck & A. C. F. Cocks), pp. 35–46. Dordrecht: Kluwer.

Bridgwater, J. 1980 On the width of failure zones. *Géotechnique* **30**, 533–536.

Butcher, A. P. & Powell, J. J. M. 1995 The effects of geological history on the dynamic stiffness in soils. In *The interplay between geotechnical engineering and engineering geology* (*Proc. 11th European Conf. on Soil Mechanics and Foundation Engineering, Copenhagen*), vol. 1, pp. 27–36. Bulletin 11, Danish Geotechnical Society.

Card, G. B. & Carder, D. R. 1993 A literature review of the geotechnical aspects of the design of integral bridge abutments. Project report 52, Transport Research Laboratory.

Chen, Y.-C. & Ishibashi, I. 1990 Dynamic shear modulus and evolution of fabric of granular materials. *Soils and Foundations* **30**(3), 1–10.

Cuccovillo, T. & Coop, M. R. 1997 Yielding and pre-failure deformation of structured sands. *Géotechnique* **47**, 491–508.

Desrues, J., Lanier, J. & Stutz, P. 1985 Localisation of the deformation in tests on sand sample. *Engng Fracture Mech.* **21**, 909–921.

Desrues, J., Chambon, R., Mokni, M. & Mazerolle, F. 1996 Void ratio evolution inside shear bands in triaxial sand specimens studied by computed tomography. *Géotechnique* **46**, 529–546.

Finno, R. J., Viggiani, G., Harris, W. W. & Mooney, M. A. 1998 Strain localisation in plane strain compression of sand. *Proc. 4th Int. Workshop, Gifu, Japan, September 1997*. Rotterdam: Balkema. (In the press.)

Gajo, A. & Muir Wood, D. 1998 A kinematic hardening constitutive model for sands: the multiaxial formulation. *Int. J. Numer. Analyt. Methods Geomech.* (In the press.)

Gavrilov, K. L. 1997 Self-organisation of uninterrupted traffic flow. In *Powders and grains 97* (ed. R. P. Behringer & J. T. Jenkins), pp. 523–526. Rotterdam: Balkema.

Gray, J. M. N. T. 1997 Granular avalanches on complex topography. In *Mechanics of granular and porous materials* (ed. N. A. Fleck & A. C. F. Cocks), pp. 275–286. Dordrecht: Kluwer.

Gudehus, G. 1996 A comprehensive constitutive equation for granular materials. *Soils and Foundations* **36**(1), 1–12.

Hallworth, M. A. & Huppert, H. E. 1998 Abrupt transitions in high-concentration, particle-driven gravity currents. *Phys. Fluids* **10**, 1083–1087.

Inagaki, H., Iai, S., Sugano, T., Yamazaki, H. & Inatomi, T. 1996 Performance of caisson type quay walls at Kobe Port. In *Soils and Foundations*, pp. 119–136. (Special issue on geotechnical aspects of the 17 January 1995 Hyogoken-Nambu earthquake.)

Kumar, G. V. & Muir Wood, D. 1997 Mechanical behaviour of mixtures of kaolin and coarse sand. In *Mechanics of granular and porous materials* (ed. N. A. Fleck & A. C. F. Cocks), pp. 57–68. Dordrecht: Kluwer.

Lambe, T. W. & Whitman, R. V. 1969 *Soil mechanics*. Wiley.

LoPresti, D. C. F., Jamiolkowski, M., Pallara, O., Cavallaro, A. & Pedroni, S. 1997 Shear modulus and damping of soils. *Géotechnique* **47**, 603–617.

Lupini, J. F., Skinner, A. E. & Vaughan, P. R. 1981 The drained residual strength of cohesive soils. *Géotechnique* **31**, 181–213.

Manzari, M. T. & Dafalias, Y. F. 1997 A critical state two-surface plasticity model for sands. *Géotechnique* **57**, 255–272.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Muir Wood, D. & Stone, K. J. L. 1994 Some observations of zones of localisation in model tests on dry sand. In *Localisation and bifurcation theory for soils and rocks* (ed. R. Chambon, J. Desrues & I. Vardoulakis), pp. 155–164. Rotterdam: Balkema.

Nichols, R. J., Sparks, R. S. J. & Wilson, C. J. N. 1994 Experimental studies of the fluidisation of layered sediments and the formation of fluid escape structures. *Sedimentol.* **41**, 233–253.

Pennington, D. S., Nash, D. F. T. & Lings, M. L. 1997 Anisotropy of G_0 shear stiffness in Gault clay. *Géotechnique* **47**, 391–398.

Roberts, A. J. 1994 *A one-dimensional introduction to continuum mechanics*. World Scientific.

Roesler, S. K. 1979 Anisotropic shear modulus due to stress anisotropy. *J. Geotech. Engng ASCE* **105**, 871–880.

Roscoe, K. H. 1970 The influence of strains in soil mechanics: 10th Rankine lecture. *Géotechnique* **20**, 129–170.

Roscoe, K. H. & Burland, J. B. 1968 On the generalised stress–strain behaviour of 'wet' clay. In *Engineering plasticity* (ed. J. Heyman & F. A. Leckie), pp. 535–609. Cambridge University Press.

Sassa, K., Fukuoka, H., Scarascia-Mugnozza, G. & Evans, S. 1996 Earthquake-induced landslides: distribution, motion and mechanisms. *Soils and Foundations*, pp. 53–64. (Special issue on geotechnical aspects of the January 17 1995 Hyogoken-Nambu earthquake.)

Scarpelli, G. & Wood, D. M. 1982 Experimental observations of shear band patterns in direct shear tests. In *Deformation and failure of granular materials* (ed. P. A. Vermeer & H. J. Luger), pp. 473–484. Rotterdam: Balkema.

Shibuya, S., Hwang, S. C. & Mitachi, T. 1997 Elastic shear modulus of soft clays from shear wave velocity measurement. *Géotechnique* **47**, 593–601.

Simpson, B., Atkinson, J. H. & Jovicic, V. 1996 The influence of anisotropy on calculations of ground settlements above tunnels. *Geotechnical aspects of underground construction in soft ground* (ed R. N. Taylor), pp. 591–595. London: Thomas Telford.

Valanis, K. C. & Read, H. E. 1982 A new endochronic plasticity model for soils. In *Soil mechanics—transient and cyclic loads* (ed. G. N. Pande & O. C. Zienkiewicz), pp. 375–417. Wiley.

Vardoulakis, I. 1978 Equilibrium bifurcation of granular earth bodies. In *Advances in analysis of geotechnical instabilities*, SM study 13, paper 3, pp. 65–119. University of Waterloo Press.

Vermeer, P. A. 1982 A simple shear-band analysis using compliances. In *Deformation and failure of granular materials* (ed. P. A. Vermeer & H. J. Luger), pp. 493–499. Rotterdam: Balkema.

Vulliet, L. 1995 Predicting large displacements of landslides. *Numerical models in geomechanics: NUMOG V* (ed. G. N. Pande & S. Pietruszczak), pp. 527–532. Rotterdam: Balkema.

Yashima, A., Oka, F., Konrad, J.-M., Uzuoka, R. & Taguchi, Y. 1997 Analysis of a progressive flow failure in an embankment of compacted till. In *Deformation and progressive failure in geomechanics: IS-Nagoya 1997* (ed. A. Asaoka, T. Adachi & F. Oka), pp. 599–604. Oxford: Pergamon.

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